## ON COMPUTATION OF SENSITIVITIES FOR 3D PLASTICITY WITH NON-LINEAR HARDENING

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Algorithmic aspects of computations of design derivatives for an elasto-plastic material, with material parameters depending on or serving as design variables, are considered, for more details see [1], [2]. The considered constitutive model is the Huber-Mises deviatoric plasticity with non-linear isotropic/kinematic hardening, one which is applicable to metals.

Two constitutive algorithms are presented to deal with the nonlinearity of the hardening functions. The first one is a direct generalization of the algorithm for linear hardening, and is based on the elastic predictor/plastic corrector scheme and the radial return on the yield surface. The other one, which is used in some parallel codes, amounts to a direct (Seidel) iteration performed on a group of tensorial equations. While the first algorithm is suitable for design differentiation, the second one is not, and to overcome this problem, we derive an equivalent scalar equation, which replaces the iteration loop and the tensorial equations. The proof that the second formulation can be equivalently transformed to a scalar form, which is much more suitable for design differentiation, is the most important result of the work.

The correctness of this equivalence is also confirmed by numerical tests run by using both non-linear algorithms. The design derivatives of both of the algorithms are provided, enabling calculations of the 'explicit' design derivative of stresses for the global sensitivity equation. Besides, the paper addresses several issues related to the implementation and testing of the DSA module; among them the concept of verification tests, both outside and inside a FE code, as well as data handling. The numerical tests used for verification of the DSA module show that,

- a) the accuracy of the analytical design derivatives is very good, as indicate comparisons with the finite difference results. If some minor differences are noticeable they are confined to the boundaries of plastic zones.
- b) the finite element formulation affects the solution and its design derivatives. We examine the results for three types of elements: the standard displacement one, the mixed u-p one, and the EAS element. The mesh convergence of the design derivatives of stresses is also checked for the aforementioned formulations, and compared with the mesh convergence of stresses.

## References

- [1] K. Wisniewski, P. Kowalczyk, E. Turska, "On computation of design derivatives for Huber-Mises plasticity with non-linear hardening", *International Journal for Numerical Methods in Engineering*, Vol.57, pp.271-300, 2003.
- [2] M. Kleiber, H. Antunez, T.D. Hien, P.M. Kowalczyk, "Parameter Sensitivity in Nonlinear Mechanics", John Wiley & Sons Ltd., Chichester, England (1997).